12.9 Exercises

1–6 ■ Find the Jacobian of the transformation.

1. $x = u + 4v, \quad y = 3u - 2v$

2. $x = u^2 - v^2, \quad y = u^2 + v^2$

3. $x = \dfrac{u}{u + v}, \quad y = \dfrac{v}{u - v}$

4. $x = \alpha \sin \beta, \quad y = \alpha \cos \beta$

5. $x = uv, \quad y = vw, \quad z = uw$

6. $x = e^{u-v}, \quad y = e^{u+v}, \quad z = e^{u+v+w}$

7–10 ■ Find the image of the set S under the given transformation.

7. $S = \{(u, v) \mid 0 \leqslant u \leqslant 3,\ 0 \leqslant v \leqslant 2\}$;
$x = 2u + 3v,\ y = u - v$

8. S is the square bounded by the lines $u = 0$, $u = 1$, $v = 0$, $v = 1$; $\quad x = v,\ y = u(1 + v^2)$

9. S is the triangular region with vertices (0, 0), (1, 1), (0, 1); $x = u^2,\ y = v$

10. S is the disk given by $u^2 + v^2 \leqslant 1$; $\quad x = au,\ y = bv$

11–16 ■ Use the given transformation to evaluate the integral.

11. $\iint_R (3x + 4y)\, dA$, where R is the region bounded by the lines $y = x$, $y = x - 2$, $y = -2x$, and $y = 3 - 2x$;
$x = \frac{1}{3}(u + v),\ y = \frac{1}{3}(v - 2u)$

12. $\iint_R (x + y)\, dA$, where R is the square with vertices (0, 0), (2, 3), (5, 1), and (3, −2); $\quad x = 2u + 3v,\ y = 3u - 2v$

13. $\iint_R x^2\, dA$, where R is the region bounded by the ellipse $9x^2 + 4y^2 = 36$; $\quad x = 2u,\ y = 3v$

14. $\iint_R (x^2 - xy + y^2)\, dA$, where R is the region bounded by the ellipse $x^2 - xy + y^2 = 2$;
$x = \sqrt{2}u - \sqrt{2/3}v,\ y = \sqrt{2}u + \sqrt{2/3}v$

15. $\iint_R xy\, dA$, where R is the region in the first quadrant bounded by the lines $y = x$ and $y = 3x$ and the hyperbolas $xy = 1$, $xy = 3$; $\quad x = u/v,\ y = v$

16. $\iint_R y^2\, dA$, where R is the region bounded by the curves $xy = 1$, $xy = 2$, $xy^2 = 1$, $xy^2 = 2$; $\quad u = xy,\ v = xy^2$. Illustrate by using a graphing calculator or computer to draw R.

17. (a) Evaluate $\iiint_E dV$, where E is the solid enclosed by the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$. Use the transformation $x = au,\ y = bv,\ z = cw$.
(b) Earth is not a perfect sphere; rotation has resulted in flattening at the poles. So the shape can be approximated by an ellipsoid with $a = b = 6378$ km and $c = 6356$ km. Use part (a) to estimate the volume of Earth.

18. Evaluate $\iiint_E x^2 y\, dV$, where E is the solid of Exercise 17(a).

19–23 ■ Evaluate the integral by making an appropriate change of variables.

19. $\iint_R xy\, dA$, where R is the region bounded by the lines $2x - y = 1$, $2x - y = -3$, $3x + y = 1$, and $3x + y = -2$

20. $\displaystyle\iint_R \frac{x + 2y}{\cos(x - y)}\, dA$, where R is the parallelogram bounded by the lines $y = x$, $y = x - 1$, $x + 2y = 0$, and $x + 2y = 2$

21. $\displaystyle\iint_R \cos\left(\frac{y - x}{y + x}\right) dA$, where R is the trapezoidal region with vertices (1, 0), (2, 0), (0, 2), and (0, 1)

22. $\iint_R \sin(9x^2 + 4y^2)\, dA$, where R is the region in the first quadrant bounded by the ellipse $9x^2 + 4y^2 = 1$

23. $\iint_R e^{x+y}\, dA$, where R is given by the inequality $|x| + |y| \leqslant 1$

24. Let f be continuous on [0, 1] and let R be the triangular region with vertices (0, 0), (1, 0), and (0, 1). Show that

$$\iint_R f(x + y)\, dA = \int_0^1 u f(u)\, du$$